New Geiger-Nuttall law of odd-Z nuclei and long-lived island beyond the stable line

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Abstract Recently, we analyze the α-decay data of even-Z nuclei and propose the new Geiger-Nuttall law where the effects of the quantum numbers of α-core relative motion are naturally embedded in the law [Physical Review C 85, 044608 (2012)].In this paper, we firstly test whether the new law without any change of parameters can be applied to the α-decays of odd-Z nuclei which are more complicated than those of even-even nuclei. Then the nuclear shell effect around N=126 is analyzed for very proton-rich nuclei with Z=85–92 based on the data of α-decay energies and half-lives. A long-lived island beyond the stable line is proposed where the half-lives of nuclei on this island are abnormally long. The mechanism of the appearance of the island and its significance to other mass ranges are discussed.

Key words New Geiger-Nuttall law, α-decay, Odd-Z nuclei, Long-lived island

1 Introduction

It is well known that β-decay is dominant for the stability of light and medium nuclei and the nuclei near the β-stable line are stable or have longer half-lives than the nuclei far from stable line[1]. For light and medium nuclei, it is also observed that the half-lives of nuclei decrease from the stable line to nuclear drip line for an isotopic chain[1]. Usually, even-even nuclei in these regions are more stable than the neighboring odd nuclei and even-even nuclei have longer half-lives than neighboring odd ones because β-decay is dominant in these regions. However, for heavier nuclei beyond ^{208}Pb (^{208}Pb is the last stable one currently), α-decay[2-8] plays an increasingly important role[9-20] because both the strong interaction and the Coulomb interaction gradually govern the nuclear stability with the increase of proton number. In some cases, spontaneous fissions also become important to heavy nuclei and superheavy nuclei. The appearance of different decay modes could change the old views of nuclear stability from researches of β-decay near the stable line. The appearance of different decay modes and their competitions will also be important to the possible existence of long-lived nuclei or long-lived islands beyond ^{208}Pb. Searching for a long-lived heavy nuclide or a new long-lived element will bring a new impact to current researches of nuclear physics.

Recently, we propose the new Geiger-Nuttall law[21] for even-Z nuclei by including the effects of quantum numbers on α-decay half-lives. In this paper, we firstly extend our researches to α-decay half-lives of odd-Z nuclei in order to test the reliability of the law for odd-Z nuclei. Secondly, we analyze the variation of total half-lives of Z=85−91 isotopic chains and explore the effect of the magic number N=126 on the stability of proton-rich nuclei far from the stable line. We will point out that a long-lived island manifests itself for these isotopes with 110≤N≤126. This behavior of abnormally long half-lives has not been observed in other mass ranges and exploration on its mechanism can be useful for future researches on other heavy nuclei far from stability.

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2 Methods and results

We start from an analytical formula of α-decay half-lives of even-even nuclei :

$$\lg T_{1/2} = a\mu^{1/2} Z_c Z_d / Q^{1/2} + b\mu^{1/2} \left(Z_c Z_d\right)^{1/2} + c. \quad (1)$$

In this formula, the values of the three parameters are a=0.39961, b=−1.31008, c=−17.00698 for even-even[7,21] nuclei. $T_{1/2}(s)$ is the half-life of α-decay and Q(MeV) is the corresponding decay energy. Z_c and Z_d arethe charge numbers of the cluster and daughter nucleus,respectively. $\mu=A_cA_d/(A_c+A_d)$ is the reduced mass and A_c, A_d are the mass numbers of the cluster and daughter nucleus, respectively. Z_c=2 and A_c=4 for α-decay. The details of this formula are given in previous publications[7,21]. This formula is called the original Geiger-Nuttall law[21] as it is a natural realization of both the Geiger-Nuttall law and the Viola-Seaborg formula[2] towards the unified description of α-decay and cluster radioactivity[7,19]. The new Geiger-Nuttall law is proposed[21] and its expression is as follows:

$$\begin{aligned}\lg T_{1/2} = a\mu^{1/2} Z_c Z_d / Q^{1/2} + b\mu^{1/2} \left(Z_c Z_d\right)^{1/2} \\ + c + S + Pl\left(l+1\right).\end{aligned} \quad (2)$$

In this formula, S is the change of radial quantum number of the α-core relative motion and l in the last term is the quantum number of angular momentum of α-particle[21]. They are the effects of quantum numbers on decay half-lives[21]. Here S=1 for N≤126 and S=0 for N≥127[21]. For favored α-decay transitions between groundstates of nuclei, l=0 is usually dominant when the ground statesof parent nucleus and daughter nucleus have the same spin andparity. Here the three parameters a, b, c have the same values as those in Eq.(1).

It is stressed that the last two terms in Eq.(2) are originated from the effect of quantum numbers of the α-decay process where a detailed explanation is given in Ref.[21]. Because the α-particle moves around the core before the decay, its motion is described by the three quantum numbers n, l, m in a central potential[21]. When the nuclei is crossed the magic number such as N=126, the quantum number can be different and S is the change of the radial number. The quantity l in the last term is the angular momentum carrying by the α-particle during the decay process[21]. For favored decay, the angular momentum l is zero.

We use both Eq.(1) and Eq.(2) to calculate α-decay half-lives of odd-Z isotopes with Z=85−91 for favored transitions. The numerical results of Z=85 and Z=87 with both equations are drawn in Figs.1 and 2.

The deviations of the logarithm of alpha-decay half-lives for At and Fr isotopes are drawn in Figs.1 and 2, where the results with Eq.(1) are denoted as "original law" and the results with Eq.(2) are denoted as "new law". It is seen clearly from Fig.1 that the deviation between calculated results with the original law and the experimental data is abnormally large for N≤126. In contrast with this, it is also seen that the deviation between calculated results with the new law and the experimental data is reasonable. Figs.1 and 2 show the same effect and this is similar to the case of even-Z nuclei. This clearly shows that the new law is valid for odd-Z isotopes without introducing any additional adjustments. The remedy achieved by the quantum effect S is also remarkable for odd-Z nuclei. Therefore Figs.1 and 2 demonstrate that the agreement between the new law and the experimental data is good although the deviation between the original law and the experimental ones is extraordinarily large for N≤126 nuclei. These are similar to the cases of even-Z nuclei and we do not repeat the discussions further. For odd-Z isotopes, there are small staggered effects on the figures due to the difference between odd-A nuclei and odd-odd nuclei.

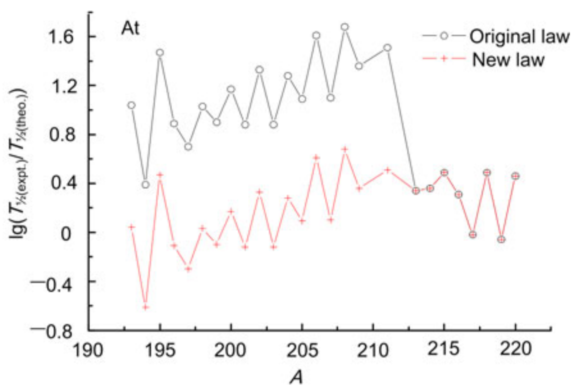

Fig.1 (Color online) Logarithms of the ratios between experimental α-decay half-lives and theoretical ones for At isotopes with the original law (Eq.(1)) and with the new law (Eq.(2)). The original law and new law go together in the range of N≥128.

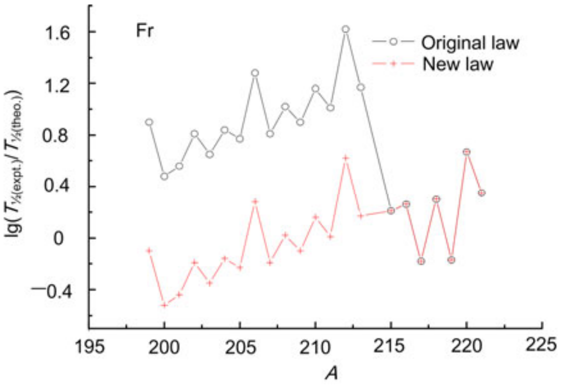

Fig.2 (Color online) Logarithms of the ratios between experimental α-decay half-lives and theoretical ones for Fr isotopes with the old law (Eq.(1) and with the new law (Eq.(2)). The original law and the new law go together in the range of $N \geq 128$.

It is seen that there are the staggered effects in Figs.1 and 2. This may be from the effect of odd nucleons. When we make numerical calculations of odd-A and odd-odd nuclei, we directly use Eq.(2) which is from the even-even nuclei and no additional adjustment is introduced for odd nuclei. Even so, new law can reach reasonable agreement with the experimental data within a factor 3. This shows the correctness of the new law. One should notice the best agreement corresponds to zero for the value of the logarithm of the ratio between experimental half-life and calculated value. In Figs.1 and 2, some from new law are above zero and others are below zero. This is a correct trend of the new law.

The numerical results of Z=85−91 isotopes with Eq.(2) are also listed in Tables 1 and 2.

Table 1 Experimental decay energies of nuclei (Q(MeV)) and the logarithms of α-decay half-lives of Z=85 and Z=87 isotopes calculated with new Geiger-Nuttall law (lg$T_{\text{theo.}}$) and experimental ones (lg$T_{\text{expt.}}$)

Nuclei	Q / MeV	lg$T_{\text{expt.}}$ / s	lg$T_{\text{theo.}}$ / s	Nuclei	Q / MeV	lg$T_{\text{expt.}}$ / s	lg$T_{\text{theo.}}$ / s
^{220}At	6.050	3.44	2.99	^{221}Fr	6.458	2.47	2.12
^{219}At	6.324	1.76	1.82	^{220}Fr	6.801	1.44	0.77
^{218}At	6.874	0.18	−0.32	^{219}Fr	7.449	−1.70	−1.53
^{217}At	7.201	−1.49	−1.47	^{218}Fr	8.014	−3.00	−3.30
^{216}At	7.950	−3.52	−3.84	^{217}Fr	8.469	−4.77	−4.60
^{215}At	8.178	−4.00	−4.49	^{216}Fr	9.175	−6.15	−6.41
^{214}At	8.987	−6.25	−6.61	^{215}Fr	9.540	−7.07	−7.27
^{213}At	9.254	−6.90	−7.25	^{213}Fr	6.905	1.54	1.37
^{211}At	5.982	4.79	4.28	^{212}Fr	6.529	3.45	2.82
^{209}At	5.757	5.68	5.32	^{211}Fr	6.660	2.32	2.30
^{208}At	5.751	6.03	5.35	^{210}Fr	6.650	2.50	2.34
^{207}At	5.872	4.88	4.78	^{209}Fr	6.777	1.75	1.85
^{206}At	5.888	5.31	4.70	^{208}Fr	6.790	1.82	1.80
^{205}At	6.020	4.20	4.11	^{207}Fr	6.900	1.19	1.38
^{204}At	6.070	4.16	3.89	^{206}Fr	6.923	1.58	1.30
^{203}At	6.210	3.16	3.28	^{205}Fr	7.055	0.59	0.82
^{202}At	6.354	3.01	2.68	^{204}Fr	7.171	0.25	0.40
^{201}At	6.473	2.08	2.20	^{203}Fr	7.260	−0.26	−0.09
^{200}At	6.596	1.88	1.71	^{202}Fr	7.389	−0.54	−0.34
^{199}At	6.780	0.91	1.01	^{201}Fr	7.520	−1.21	−0.78
^{198}At	6.893	0.62	0.60	^{200}Fr	7.620	−1.62	−1.10
^{197}At	7.100	−0.44	−0.14	^{199}Fr	7.810	−1.80	−1.70
^{196}At	7.200	−0.60	−0.48	–	–	–	–
^{195}At	7.339	−0.48	−0.95	–	–	–	–
^{194}At	7.291	−1.40	−0.79	–	–	–	–
^{193}At	7.490	−1.40	−1.44	–	–	–	–

Note: The experimental branching ratios of α-decay in ^{196}At and ^{202,203}Fr are unknown and we assume that they are 100% for α-decay.

Table 2 Experimental decay energies of nuclei (Q(MeV)) and the logarithms of α-decay half-lives of Z=89 and Z=91 isotopes calculated with new Geiger-Nuttall law (lg$T_{\rm theo.}$) and experimental ones (lg$T_{\rm expt.}$)

Nuclei	Q / MeV	lg$T_{\rm expt.}$ / s	lg$T_{\rm theo.}$ / s	Nuclei	Q / MeV	lg$T_{\rm expt.}$ / s	lg$T_{\rm theo.}$ / s
^{227}Ac	5.042	10.70	10.12	^{231}Pa	5.150	12.01	11.44
^{225}Ac	5.935	5.94	5.31	^{229}Pa	5.835	7.43	6.72
^{223}Ac	6.783	2.10	1.66	^{227}Pa	6.580	3.43	3.31
^{222}Ac	7.137	0.70	0.33	^{226}Pa	6.987	2.16	1.69
^{221}Ac	7.780	−1.28	−1.85	^{225}Pa	7.390	0.23	0.21
^{219}Ac	8.83	−4.93	−4.88	^{224}Pa	7.694	−0.07	−0.82
^{218}Ac	9.380	−5.97	−6.26	^{223}Pa	8.330	−2.29	−2.80
^{217}Ac	9.832	−7.16	−7.31	^{221}Pa	9.25	−5.23	−5.30
^{215}Ac	7.744	−0.77	−0.74	^{220}Pa	9.83	−6.11	−6.69
^{214}Ac	7.350	0.96	0.57	^{219}Pa	10.08	−7.28	−7.25
^{213}Ac	7.50	−0.14	0.06	^{217}Pa	8.489	−2.46	−2.27
^{212}Ac	7.52	−0.04	−0.01	^{216}Pa	8.097	−0.98	−1.11
^{211}Ac	7.62	−0.67	−0.34	^{215}Pa	8.240	−1.85	−1.54
^{210}Ac	7.61	−0.46	−0.31	^{214}Pa	8.27	−1.77	−1.63
^{209}Ac	7.73	−1.04	−0.70	^{213}Pa	8.39	−2.15	−1.98
^{208}Ac	7.73	−1.01	−0.70	^{212}Pa	8.43	−2.10	−2.10
^{207}Ac	7.84	−1.51	−1.05	^{211}Pa	8.53*	–	−2.39
^{206}Ac	7.94	−1.60	−1.36	^{210}Pa	8.57*	–	−2.50
^{205}Ac	8.04*	–	−1.67	^{209}Pa	8.67*	–	−2.78

Note: *, the decay energies of ^{205}Ac and $^{209\text{-}211}$Pa are unknown and we estimated their values.Some nuclei are missing because their α-decays are not observed or their branching ratios of α-decay are too small (≤ 0.1%) or their decays belong to hindered transitions.

In Table 1, the first column denotes the parent nucleus and the second column represents the α-decay energy of the nucleus. The third column and the fourth column represent the logarithm of experimental α-decay half-life and the logarithm of theoretical half-life, respectively. The experimental data are from the nuclear mass table by Audi *et al*.[22,23] In Tables 1 and 2, some nuclei are missing on the isotopic chain as there are no α-decay data on them or some data are uncertain due to very small branching of α-decay[22,23]. The hindered transitions of odd-Z isotopes with the change of angular momentum or parity such as N=127 are not included in this paper and we made researches on them for even-Z cases in our previous paper[21]. The columns 5−8 have similar meanings as those of columns 1−4. The results of odd-Z nuclei with Eq.(2) are not perfect compared with those of even-even nuclei in our previous publication. This is expected because the ground-state transitions of even-even nuclei are simple and can be easily treated in theory. For odd-Z nuclei, odd-nucleon can complicate the transition of α-decay and this can bring inaccuracy for both experimental measurements of decay half-lives and theoretical calculations of half-lives compared with the cases of even-even nuclei[22-24]. Although we do not introduce any additional adjustment to the calculation of odd-Z nuclei in this paper, the results with Eq.(2) for odd-Z nuclei are good and this confirms the validity of new Geiger-Nuttall law for odd-Z nuclei.The good agreement between the data and theoretical results is reached and this can be clearly seen from Tables 1 and 2. Now let us discuss in detail the results of At (Z=85) isotopes in the columns 1−4 of Table 1. For many nuclei of At isotopes, the calculated half-lives are in agreement with the data within a factor of 1−3 (the corresponding deviation of logarithm is 0−0.5) and only for a few nuclei the agreement between the calculated results and the data is approximately a factor of 4−5 (the corresponding deviation of logarithm is 0.6−0.7). For Fr isotopes in Table 1, the calculated half-lives also agree with the data well and this confirms the validity of new Geiger-Nuttall law for odd-Z nuclei. The numerical results of Ac (Z=89) and Pa (Z=91) isotopes are listed in Table 2 and it confirms again that the reliability of the new law for odd-Z isotopes according to the good

agreement between calculated values and experimental data. For the nuclei on Pa isotopic chains, ^{231}Pa is very special[22-24]. Although the spin and parity of its ground state are the same as its daughter nucleus ^{227}Ac, the α-decay branching ratio to the ground state of ^{227}Ac is approximately 11%. This shows again the complexity of the decay for odd-*Z* nuclei compared with the cases of even-even nuclei. For some proton-rich nuclei such as $^{208\text{-}210,212\text{-}213,217}$Ac, the branching ratio of α-decay is unknown and we assume it is 100% according to information of neighboring nuclei[22,23].For $^{209\text{-}211}$Pa and ^{205}Ac, their experimental decay energies and half-lives are unknown and the experiments on these will be carried out in the Institute of Modern Physics at Lanzhou in China. We estimate their decay energies (denoted with a star in Table 2) according to the trend of decay energies and predict their half-lives by the new law with the inputs of the estimated decay energies. These will be compared with the measured half-lives and decay energies in the Institute of Modern Physics at Lanzhou in China.

Before ending this paper, it is interesting to discuss the stability of nuclei for some nuclei far from the stable line and to investigate the effect of *N*=126 shell closure on the half-lives of proton-rich nuclei. According to textbooks of modern physics and nuclear physics, the *β*-stable line lies approximately on *N*=*Z* for very light nuclei and *N*=1.54*Z* for heavy nuclei around ^{208}Pb. For heavy nuclei such as ^{208}Pb, ^{232}Th and ^{238}U, they are stable or have very long half-lives because they approximately lie on the stable line. When it goes gradually away from the stable line, the nuclear half-life will decrease on an isotopic chain. This is well known for light and heavy nuclei with *Z*≤82. However, it is observed that the total nuclear half-lives with *N*≤126 on *Z*=85−92 isotopic chains are extraordinarily long due to the sudden decrease of the decay energies at the shell closure. This clearly shows that the magic number *N*=126 exists for proton-rich region with *Z*=85−92 although some magic numbers can disappear when it comes to light neutron-rich nuclei. It is the existence of the magic number *N*=126 in this region that leads to the appearance of the long-lived island for *Z*=85−92 nuclei and we draw it for some odd-*Z* nuclei in Fig.3.

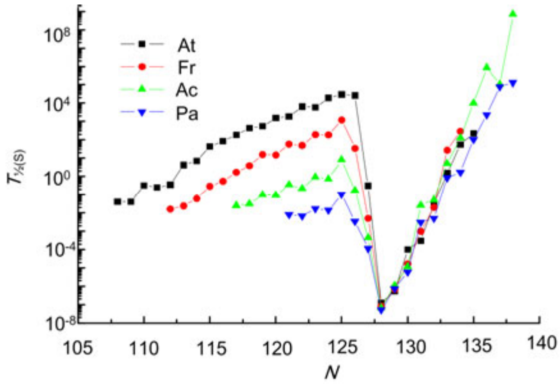

Fig.3 (Color online) Variation of experimental total half-lives of *Z*=85−91 isotopes towards the proton-rich side. The total half-lives on an isotopic chain become shorter with the decrease of neutron number when getting away from the stable line. The long-lived island appears for nuclei with *N*≤126.

From Fig.3, it is seen that total nuclear half-lives of *Z*=85−91 decrease rapidly from *N*=138 to *N*=128 when it goes to proton-rich side from the stable line. The shortest half-life reaches around *N*=128 for each isotopic chain due to the appearance of the maximum decay energy for *α*-transitions in ground states. Then nuclear half-lives increase rapidly from *N*=128 to *N*=126 and reach a local maximum around *N*=125 or *N*=126. After that, the nuclear half-lives decrease very slowly and this leads to the formation of an island with longer half-lives beyond the traditional stable line. It is well known that even-even nuclei are more stable than their neighboring odd-*A* nuclei and odd-*A* nuclei are more stable than their neighboring odd-odd nuclei when *β*-decay is dominant for nuclear decays. However, for *α*-decay, odd-*A* nuclei on an even-*Z* isotopic chain can usually have longer half-lives than their neighboring even-even nuclei because of quantum blocking effect of odd-nucleon for *α*-decay. Odd-odd nuclei on an odd-*Z* chain can also have longer half-lives than their neighboring odd-*A* nuclei due to the same effect for *α*-decay. Therefore, researches on *α*-decay in heavy-mass region can change the old view of nuclear stability from researches on nuclei near the stable line. Similar phenomena on half-lives of even nuclei and odd nuclei are also observed for spontaneous fission half-lives of heavy nuclei. Now let us turn to the scope of this island. At present, according to available data of nuclei, we do not know the upper limit of proton number for this island and we suggest that more experiments on

Z=92−94 chains be done in order to explore the upper limit of this island. In the future, it will be interesting to explore further mechanism of this kind of islands and to search for new islands far from the stable line as they are directly related to both the existence of magic numbers and the saturation of nuclear forces near the drip line. This could be helpful for the search of other spherical islands beyond ^{208}Pb.

3 Conclusion

In summary, we calculate the half-lives of Z=85−91 nuclei with new Geiger-Nuttall law and test the validity of new Geiger-Nuttall law for odd-Z isotopic chains. It is discovered that the new law can be applied to odd-Z nuclei without additional introduction of new parameters. A new island with abnormally long half-lives manifests itself for Z=85−92 isotopes with $N\leq 126$ due to the spherical shell closure. This is the first island with abnormally long half-lives beyond the stable line. The mechanism of the appearance of this island could be used to explore other long lifetime islands beyond the stable line. It is also useful for investigating the variation of magic numbers for nuclei far from stability.

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